Intuition and Proof Sketch

**Problem Statement**

Let $v_n$ be a distribution on $R$. Let $f_0$ be the known parametric distribution of test statistics with effect size $e$, e.g. $f_0 = N(m, 1)$. Draw $\{x_i\}_{i=1}^n \sim v_n$ (unobserved) and $\{X_i\}_{i=1}^n \sim f_0$ (observed).

Goal

Estimate, for all $\gamma \in R$ and without overestimating,

$$
\zeta_n(\gamma) := P_{v_n}(v > \gamma)
$$

Our Answer

Yes. If we are testing $n$ hypotheses and the effect sizes are small, then we can detect the existence of discoveries with $n$ times fewer samples than we would need for identification.

**Theoretical Results**

**Theorem.** For $i = 1, 2, \ldots, n$, let $v_i \sim v_n$ and $X_i \sim f_0$, where each draw is i.i.d. Let our simultaneous estimator be given by (1), then

$$
P(\exists \gamma: \zeta_n(\gamma) > \zeta_n(\gamma)) \leq \alpha.
$$

Furthermore, with probability at least $1 - \delta$, for all $\gamma \in R$ and $\epsilon \in (0, \zeta_n(\gamma))$, we have $\zeta_n(\gamma) - \zeta_n(\gamma) \leq \epsilon$ whenever

$$
\epsilon \geq \frac{\log \left( \frac{4}{\delta} \right)}{\min_{\gamma: v_i(\gamma) \leq \zeta_n(\gamma) - \epsilon} ||F_i - F_0||_w^2}.
$$

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By the DKW inequality, the true CDF $F_n$ is contained in the $\epsilon_n$ ball around $F_0$. If we find the $v$ that stays inside this ball but has the least mass above $\gamma$ (the minimizer of (1), shown by $F_{\text{min}}$), then with high probability, this is a lower bound on $\zeta_n(\gamma)$.

The sample complexity result follows from the DKW inequality and the triangle inequality. We want to bound $||F_n - F_0||_w$ subject to the constraints on $v$ in (1), so we control it using the DKW bound on $||F_n - F_{\text{min}}||_w$ and the constrained quantity $||F_n - F_{\text{min}}||_w$ found in the theorem statement.